

Effects of Electromagnetic Field on Energy Density Inhomogeneity in Self-Gravitating Fluids

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Abstract

This paper is devoted to study the effects of electromagnetic field on the energy density inhomogeneity in the relativistic self-gravitating fluids for spherically symmetric spacetime. Two important equations of the Weyl tensor are formulated which help to analyze the energy density inhomogeneity in this scenario. We investigate two types of fluids, i.e., non-dissipative and dissipative. The non-dissipative fluid further includes dust, locally isotropic, and locally anisotropic charged fluids. We explore the effects of different factors on energy density inhomogeneity in all these cases, in particular, the effect of charge.

Keywords: Energy density inhomogeneity; Electromagnetic field; Weyl tensor; Bianchi identities and Transport equation.

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1 Introduction

Gravitational collapse emerges from a highly inhomogeneous initial state which can be explained in terms of inhomogeneous energy density distribution. Energy density inhomogeneity plays a vital role in the collapse of

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self-gravitating fluid. Penrose [1] discussed the role of the Weyl tensor in the evolution of self-gravitating system. He provided a simple relation between the Weyl tensor and energy density to investigate the gravitational arrow of time. The Weyl tensor may be represented exclusively in terms of energy density and local anisotropy of pressure, which affects the fate of gravitational collapse.

There is a large body of literature available [2]-[6] which indicate the importance of energy density inhomogeneity in self-gravitating fluid. Joshi and Dwivedi [2] analyzed the Tolman-Bondi model to examine the nature and occurrence of naked singularities for inhomogeneous gravitational collapse. Triginer and Pavon [3] discussed heat transport equation in an inhomogeneous spherically symmetric universe. Mena and Tavokol [4] investigated how it is important in the dust collapse. Herrera et al. [5] derived a relation for the active gravitational mass of collapsing fluid distribution to explain the effects of density inhomogeneity and local anisotropy on spherical collapse. The same authors [6] also studied the behavior of locally anisotropic, self-gravitating spherical symmetric dissipative fluid with the Weyl tensor and density inhomogeneity.

The study of self-gravitating spherically symmetric charged fluid distribution has been the subject of interest for many people. Rosseland [7] and Edington [8] started the study of self-gravitating spherically symmetric charged fluid distribution. Since then, a lot of work has been done to investigate the effects of electric charge on the structure and evolution of self-gravitating system [9]-[16]. Di Prisco et al. [17] investigated the non-adiabatic charged spherically symmetric gravitational collapse as well as the energy density inhomogeneity. Sharif and his collaborators [18] derived dynamical as well as transport equations of matter dissipating in the form of shear viscosity to see the effect of charge on gravitational collapse.

The gravitational collapse is highly dissipative process [19, 20], hence its effects are much important in the study of collapse. Misner [21] explained the dissipative relativistic gravitational collapse by using the streaming out approximation. Two possible approximations (diffusion and streaming out) are usually considered in dissipative process. Diffusion approximation is the approximation for which energy flux of radiation (like thermal conduction) is proportional to the gradient of temperature. Israel and Stewart [22] formulated transport equation required for diffusion approximation. Lattimer [23] explained that during emission process, the role of radiation transport is closer to diffusion approximation than the streaming out approximation. Re-

cently, Herrera [24] explored energy density inhomogeneity in self-gravitating fluid as well as its stability.

In this paper, we extend this work to see the effects of charge on energy density inhomogeneity due to fluid distribution. We shall discuss the effects of electric charge on the relationship between the Weyl tensor and energy density inhomogeneity. The outline of the paper is as follows. In the next section, we review the relevant kinematics and Einstein-Maxwell field equations for a spherically symmetric distribution of collapsing charged fluid. Section 3 is devoted to dynamical as well as transport equations and two equations of the Weyl tensor. In section 4, different aspects of fluid are considered to explore energy density inhomogeneity. The last section contains the conclusion of the results.

2 Spacetime and Matter Distribution

We take spherically symmetric distribution of dissipative collapsing charged fluid bounded by a spherical surface Σ in the comoving coordinates as

$$ds^2 = -A^2 dt^2 + B^2 dr^2 + C^2(d\theta^2 + \sin^2 \theta d\phi^2), \quad (1)$$

where A , B and C are functions of t and r . Matter under consideration is anisotropic fluid suffering dissipation in the form of heat, i.e.,

$$T_{\alpha\beta}^{(m)} = (\mu + P_{\perp})V_{\alpha}V_{\beta} + P_{\perp}g_{\alpha\beta} + (P_r - P_{\perp})\chi_{\alpha}\chi_{\beta} + q_{\alpha}V_{\beta} + V_{\alpha}q_{\beta} + \epsilon l_{\alpha}l_{\beta}, \quad (2)$$

where μ , P_r , P_{\perp} , V^{α} , χ^{α} , q^{α} , l^{α} and ϵ represent the energy density, the radial pressure, the tangential pressure, the four velocity of the fluid, a unit four-vector along the radial direction, the heat flux, a radial null four-vector and the energy density of null fluid describing dissipation in the free streaming approximation respectively. These quantities satisfy

$$V^{\alpha}V_{\beta} = -1, \quad V^{\alpha}q_{\beta} = 0, \quad \chi^{\alpha}\chi_{\beta} = 1, \quad l^{\alpha}V_{\alpha} = -1, \quad \chi^{\alpha}V_{\alpha} = 0, \quad l^{\alpha}l_{\alpha} = 0.$$

Also, we define

$$V^{\alpha} = A^{-1}\delta_0^{\alpha}, \quad q^{\alpha} = qB^{-1}\delta_1^{\alpha}, \quad l^{\alpha} = A^{-1}\delta_0^{\alpha} + B^{-1}\delta_1^{\alpha}, \quad \chi^{\alpha} = B^{-1}\delta_1^{\alpha}.$$

The shear tensor is given by

$$\sigma_{\alpha\beta} = V_{(\alpha;\beta)} + a_{(\alpha}V_{\beta)} - \frac{1}{3}\Theta h_{\alpha\beta},$$

where $h_{\alpha\beta} = g_{\alpha\beta} + V_\alpha V_\beta$. The four acceleration a_α and the expansion Θ are

$$a_\alpha = V_{\alpha;\beta} V^\beta, \quad \Theta = V^\alpha{}_{;\alpha},$$

which yield

$$a_1 = \frac{A'}{A}, \quad a^2 = a^\alpha a_\alpha = \left(\frac{A'}{AB} \right)^2, \quad \Theta = \frac{1}{A} \left(\frac{\dot{B}}{B} + 2 \frac{\dot{C}}{C} \right). \quad (3)$$

The non-zero components of the shear tensor will be

$$\sigma_{11} = \frac{2}{3} B^2 \sigma, \quad \sigma_{22} = \frac{\sigma_{33}}{\sin^2 \theta} = -\frac{1}{3} C^2 \sigma,$$

with its scalar

$$\sigma^{\alpha\beta} \sigma_{\alpha\beta} = \frac{2}{3} \sigma^2, \quad \sigma = \frac{1}{A} \left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C} \right). \quad (4)$$

The electromagnetic energy-momentum tensor is

$$T_{\alpha\beta}^{(em)} = \frac{1}{4\pi} (F_\alpha^\gamma F_{\beta\gamma} - \frac{1}{4} F^{\gamma\delta} F_{\gamma\delta} g_{\alpha\beta}), \quad (5)$$

where $F_{\alpha\beta}$ is the electromagnetic field tensor. In terms of four-vector formulation, the Maxwell field equations can be written in the form

$$F_{\alpha\beta} = \phi_{\beta,\alpha} - \phi_{\alpha,\beta}, \quad F_{;\beta}^{\alpha\beta} = \mu_0 J^\alpha, \quad (6)$$

where ϕ_α , J^α and μ_0 represent the four potential, four current and magnetic permeability, respectively. Since charge is considered at rest in comoving coordinates, so J^α and ϕ_α become

$$\phi_\alpha = \phi \delta_\alpha^0, \quad J^\alpha = \rho V^\alpha.$$

Here the charge density ρ and electric scalar potential ϕ are both functions of t and r . The charge conservation equation gives

$$s(r) = 4\pi \int_0^r \rho B C^2 dr, \quad (7)$$

where $s(r)$ is the electric charge interior to radius r . For metric (1), the Maxwell equations turn out to be

$$\phi'' - \left(\frac{A'}{A} + \frac{B'}{B} - 2\frac{C'}{C} \right) \phi' = 4\pi\rho AB^2, \quad (8)$$

$$\dot{\phi}' - \left(\frac{\dot{A}}{A} \frac{\dot{B}}{B} - 2\frac{\dot{C}}{C} \right) \phi' = 0. \quad (9)$$

Equation (8) yields

$$\phi' = \frac{sAB}{C^2}, \quad (10)$$

which satisfies Eq.(9). The field equations for charged dissipative fluid, $G_{\alpha\beta} = 8\pi(T_{\alpha\beta}^{(m)} + T_{\alpha\beta}^{(em)})$, are [17]

$$\begin{aligned} 8\pi(T_{00}^{(m)} + T_{00}^{(em)}) &= 8\pi(\mu + \epsilon)A^2 + \frac{(sA)^2}{C^4} \\ &= \left(2\frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) \frac{\dot{C}}{C} - \left(\frac{A}{B} \right)^2 \\ &\times \left[2\frac{C''}{C} + \left(\frac{C'}{C} \right)^2 - 2\frac{B'C'}{BC} - \left(\frac{B}{C} \right)^2 \right], \end{aligned} \quad (11)$$

$$\begin{aligned} 8\pi(T_{01}^{(m)} + T_{01}^{(em)}) &= -8\pi(q + \epsilon)AB \\ &= -2 \left(\frac{\dot{C}'}{C} - \frac{\dot{B}C'}{BC} - \frac{\dot{C}A'}{CA} \right), \end{aligned} \quad (12)$$

$$\begin{aligned} 8\pi(T_{11}^{(m)} + T_{11}^{(em)}) &= 8\pi(P_r + \epsilon)B^2 - \frac{s^2B^2}{C^4} \\ &= -\left(\frac{B}{A} \right)^2 \left[2\frac{\ddot{C}}{C} - \left(2\frac{\dot{A}}{A} - \frac{\dot{C}}{C} \right) \frac{\dot{C}}{C} \right] \\ &+ \left(2\frac{A'}{A} + \frac{C'}{C} \right) \frac{C'}{C} - \left(\frac{B}{C} \right)^2, \end{aligned} \quad (13)$$

$$\begin{aligned}
8\pi(T_{22}^{(m)} + T_{22}^{(em)}) &= \frac{8\pi}{\sin^2 \theta} (T_{33}^{(m)} + T_{33}^{(em)}) = 8\pi P_{\perp} C^2 + \frac{s^2}{C^2} \\
&= \left(\frac{C}{A}\right)^2 \left[\frac{\dot{A}}{A} \left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) - \frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} - \frac{\dot{B}\dot{C}}{BC} \right] \\
&+ \left(\frac{C}{B}\right)^2 \left[\frac{A''}{A} + \frac{C''}{C} - \frac{A'B'}{AB} + \left(\frac{A'}{A} - \frac{B'}{B} \right) \frac{C'}{C} \right]. \quad (14)
\end{aligned}$$

3 Dynamical and Transport Equations

Using the Misner-Sharp definition [25], the mass function in the presence of charge is given by

$$m = \frac{C^3}{2} R_{23}{}^{23} + \frac{s^2}{2C} = \frac{C}{2} \left[\left(\frac{\dot{C}}{A} \right)^2 - \left(\frac{C'}{B} \right)^2 + 1 \right] + \frac{s^2}{2C}. \quad (15)$$

The proper time and radial derivatives are defined by [17]

$$D_T = \frac{1}{A} \frac{\partial}{\partial t}, \quad D_R = \frac{1}{C'} \frac{\partial}{\partial r}, \quad (16)$$

where R stands for proper areal radius. Using this definition, the velocity of the collapsing fluid is $U = D_T C = \frac{\dot{C}}{A} < 0$. Thus we can write Eq.(15) as

$$E = \frac{C'}{B} = \sqrt{1 + U^2 - \frac{2m}{C} + \left(\frac{s}{C} \right)^2}. \quad (17)$$

Define $\tilde{\mu} = \mu + \epsilon$, $\tilde{P}_r = P_r + \epsilon$, $\tilde{q} = q + \epsilon$. The rate of change of mass is

$$D_T m = -4\pi \left[\tilde{P}_r U + \tilde{q} E \right] C^2, \quad D_R m = 4\pi \left[\tilde{\mu} + \tilde{q} \frac{U}{E} \right] C^2 + \frac{s}{C} D_R s. \quad (18)$$

This shows how mass is affected by different quantities. Integration of the second equation yields

$$m = \int_0^r \left[4\pi \left(\tilde{\mu} + \tilde{q} \frac{U}{E} \right) C^2 C' + \frac{ss'}{C} \right] dr. \quad (19)$$

The contracted Bianchi identities, $(T^{(m)\alpha\beta} + T^{(em)\alpha\beta})_{;\beta} = 0$, yield

$$\begin{aligned} (T^{(m)\alpha\beta} + T^{(em)\alpha\beta})_{;\beta} V_\alpha &= \dot{\mu} + (\tilde{\mu} + \tilde{P}_r) \frac{\dot{B}}{B} + 2(\tilde{\mu} + \tilde{P}_\perp) \frac{\dot{C}}{C} \\ &+ \frac{q' A}{B} + 2\tilde{q} \frac{(AC)'}{BC}, \end{aligned} \quad (20)$$

$$\begin{aligned} (T^{(m)\alpha\beta} + T^{(em)\alpha\beta})_{;\beta} \chi_\alpha &= \dot{q} + \frac{(\tilde{P}_r)' A}{B} + 2\tilde{q} \left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) \\ &+ (\tilde{\mu} + \tilde{P}_r) \frac{A'}{B} + 2\Pi \frac{AC'}{BC} - \frac{ss' A}{4\pi BC^4} = 0, \end{aligned} \quad (21)$$

where $\Pi = \tilde{P}_r - P_\perp$. These are called dynamical equations.

The Weyl tensor is defined by

$$\begin{aligned} C_{\alpha\beta\mu}^\rho &= R_{\alpha\beta\mu}^\rho - \frac{1}{2} R_{\beta\mu}^\rho g_{\alpha\mu} + \frac{1}{2} R_{\alpha\beta}^\rho \delta_\mu^\rho - \frac{1}{2} R_{\alpha\mu}^\rho \delta_\beta^\rho + \frac{1}{2} R_{\mu}^\rho g_{\alpha\beta} \\ &+ \frac{1}{6} R (\delta_\beta^\rho g_{\alpha\mu} - g_{\alpha\beta} \delta_\mu^\rho). \end{aligned} \quad (22)$$

The electric part of the Weyl tensor, $E_{\alpha\beta} = C_{\alpha\mu\beta\nu} V^\mu V^\nu$, gives

$$E_{11} = \frac{2}{3} B^2 \varepsilon, \quad E_{22} = -\frac{1}{3} C^2 \varepsilon, \quad E_{33} = E_{22} \sin^2 \theta,$$

where

$$\begin{aligned} \varepsilon &= \frac{1}{2A^2} \left[\frac{\ddot{C}}{C} - \frac{\ddot{B}}{B} - \left(\frac{\dot{C}}{C} - \frac{\dot{B}}{B} \right) \left(\frac{\dot{A}}{A} + \frac{\dot{C}}{C} \right) \right] \\ &+ \frac{1}{2B^2} \left[\frac{A''}{A} - \frac{C''}{C} + \left(\frac{B'}{B} + \frac{C'}{C} \right) \left(\frac{C'}{C} - \frac{A'}{A} \right) \right] - \frac{1}{2C^2}, \end{aligned} \quad (23)$$

while its magnetic part vanishes due to spherical symmetry. We may also write $E_{\alpha\beta}$ as

$$E_{\alpha\beta} = \varepsilon (\chi_\alpha \chi_\beta - \frac{1}{3} h_{\alpha\beta}). \quad (24)$$

Solving Eqs.(11)-(13) with Eqs.(15) and (23), we get

$$\frac{3m}{C^3} - \frac{2s^2}{C^4} = 4\pi(\tilde{\mu} - \Pi) - \varepsilon. \quad (25)$$

Using Eqs.(18) and (23), we can write

$$[\varepsilon - 4\pi(\tilde{\mu} - \Pi)] = 3\frac{\dot{C}}{C} \left[4\pi(\tilde{\mu} + P_{\perp}) - \frac{2s^2}{3C^4} - \varepsilon \right] + 12\pi\tilde{q}\frac{AC'}{BC}, \quad (26)$$

$$[\varepsilon - 4\pi(\tilde{\mu} - \Pi)]' = -3\frac{C'}{C} \left[\varepsilon + 4\pi\Pi + \frac{2s^2}{3C^4} \right] - 12\pi\tilde{q}\frac{\dot{C}B}{AC} + \frac{ss'}{C^4}. \quad (27)$$

These equations yield a relationship between the Weyl tensor, energy density and charge that help to discuss energy density inhomogeneities given in the next section.

The corresponding transport equation for heat flux derived from Muller-Israel-Stewart theory [18, 26] is given as

$$\tau h^{\alpha\beta} V^{\gamma} q_{\beta;\gamma} + q^{\alpha} = -\kappa h^{\alpha\beta} (T_{;\beta} + Ta_{\beta}) - \frac{1}{2}\kappa T^2 \left(\frac{\tau V^{\beta}}{\kappa T^2} \right)_{;\beta} q^{\alpha}, \quad (28)$$

where κ , T and τ represent thermal conductivity, temperature and relaxation time, respectively. Due to symmetry, the above equation has only one independent component given by

$$\tau\dot{q} = -\frac{1}{2}\kappa q T^2 \left(\frac{\tau}{\kappa T^2} \right) - \frac{1}{2}\tau q \Theta A - \frac{\kappa}{B}(TA)' - qA. \quad (29)$$

For $\tau = 0$, Eckart-Landau [27] equation is recovered. In case of truncated version of the theory, the last term in Eq.(28) is removed and finally, we get

$$\tau\dot{q} = -\frac{\kappa}{B}(TA)' - qA, \quad (30)$$

which is the same as given in [24]. This shows that the electromagnetic field does not affect the heat transport equation.

4 Energy Density Inhomogeneity

In this section, we consider different aspects of fluid distribution that are responsible for energy density inhomogeneity .

4.1 Non-dissipative Charged Dust

Firstly, the case of non-dissipative charged dust is taken, i.e., $q = P_r = P_\perp = \epsilon = 0$. Since the fluid is moving along geodesic (i.e., $a^\alpha = 0 = a$), so Eq.(3) leads to $A = 1$. Consequently, Eqs.(26) and (27) reduce to

$$(\varepsilon - 4\pi\mu)^\cdot + 3\frac{\dot{C}}{C} \left(\varepsilon - 4\pi\mu + \frac{2s^2}{3C^4} \right) = 0, \quad (31)$$

$$(\varepsilon - 4\pi\mu)' = -3\frac{C'}{C}\varepsilon - 2\frac{s^2C'}{C^5} + \frac{ss'}{C^4}. \quad (32)$$

For $s = 0$, this case reduces to the uncharged case [24]. Equation (32) implies that for $\varepsilon = 0 = s$, we get $\mu' = 0$. This shows that energy density inhomogeneity depends not only on the Weyl tensor but also on charge s . For $\varepsilon = 0 = \mu'$ (i.e., when spacetime is conformally flat and energy density vanishes), it follows from Eq.(32) that $C^2(t, r) = s(r)\phi(t)$, where $\phi(t)$ is an arbitrary function. When $\mu' = 0$, we obtain

$$\varepsilon = \frac{1}{C^3} \int_0^r \left(\frac{ss'}{C} - \frac{2s^2C'}{C^2} \right) dr, \quad (33)$$

where integration function is chosen such that $\varepsilon(t, 0) = 0$. This equation shows that homogeneity in energy density implies the existence of ε in the presence of electromagnetic field.

Next, we make use of Eqs.(4) and (20) in (31) so that

$$\varepsilon = -\frac{4\pi}{C^3} \int_0^t \mu\sigma C^3 dt + \frac{2s^2}{C^4}. \quad (34)$$

Here integration function is chosen as $\varepsilon(0, r) = 0$. With the help of Raychaudhuri equation and the field equations, we get an evolution equation for shear (see [?] for detail) given by

$$\dot{\sigma} + \frac{\sigma^2}{3} + 2\frac{\Theta\sigma}{3} = -\varepsilon. \quad (35)$$

Equations (34) and (35) show that conformal flatness and shearfree conditions do not imply each other. In this case, the shearfree implies vanishing of the Weyl tensor but converse is not true.

4.2 Locally Isotropic Non-dissipative Charged Fluid

This case corresponds to a non-dissipative charged isotropic fluid, i.e., $\Pi = q = \epsilon = 0$, $P_r = P_\perp = P$. Equations (26) and (27) reduce to

$$(\varepsilon - 4\pi\mu)^\cdot + 3\frac{\dot{C}}{C} \left(\varepsilon - 4\pi(\mu + P) + \frac{2s^2}{3C^4} \right) = 0, \quad (36)$$

$$(\varepsilon - 4\pi\mu)' = -3\frac{C'}{C}\varepsilon - 2\frac{s^2 C'}{C^5} + \frac{ss'}{C^4}. \quad (37)$$

Equation (37) is exactly the same as (32) and hence the same behavior for $\varepsilon = q = 0$, $\varepsilon = \mu' = 0$ and $\mu' = 0$. Using Eqs.(4) and (20) in (36), we have

$$\varepsilon = -\frac{4\pi}{C^3} \int_0^t [(\mu + P)A\sigma C^3]dt + \frac{2s^2}{C^4}, \quad (38)$$

where $\varepsilon(0, r) = 0$. For non-dissipative locally isotropic fluid (using Raychaudhuri equation as well field equations as in the non-dissipative charged case), the evolution equation of the shear takes the form

$$\varepsilon = \frac{a'}{B} - \frac{\dot{\sigma}}{A} + a^2 - \frac{\sigma^2}{3} - \frac{2}{3}\Theta\sigma - a\frac{C'}{BC}. \quad (39)$$

It is important to mention here that neither the vanishing of the Weyl tensor implies shearfree nor the shearfree condition corresponds to conformally flatness (i.e., the Weyl tensor disappears). If we assume the fluid to be shearfree, then Eq.(38) yields

$$\varepsilon = \frac{2s^2}{C^4}. \quad (40)$$

This shows that under the effect of electromagnetic field for initial homogeneous configuration, i.e, $\varepsilon(0, r) = 0$, the Weyl tensor does not disappear at any time t .

4.3 Locally Anisotropic Non-dissipative Charged Fluid

In this case, we consider the role of pressure anisotropy with $q = \epsilon = 0$ but $\Pi \neq 0$. Under these conditions, Eqs.(26) and (27) become

$$(\varepsilon - 4\pi\mu + 4\pi\Pi)^\cdot + 3\frac{\dot{C}}{C} \left(\varepsilon - 4\pi(\mu + P_\perp) + \frac{2s^2}{3C^4} \right) = 0, \quad (41)$$

$$(\varepsilon - 4\pi\mu + 4\pi\Pi)' + 3\frac{C'}{C} \left(\varepsilon + 4\pi\Pi + \frac{2s^2}{3C^4} \right) - \frac{ss'}{C^4} = 0. \quad (42)$$

In the case of locally anisotropic fluid, the quantity $\varepsilon + 4\pi\Pi$ plays an important role instead of the Weyl tensor. The existence of the energy density inhomogeneity depends on $\varepsilon + 4\pi\Pi$ as well as charge unlike previous cases. From Eq.(42), it follows that for $\varepsilon + 4\pi\Pi = 0 = s$, we get $\mu' = 0$. When we take $\varepsilon + 4\pi\Pi = 0 = \mu'$, then Eq.(42) reduces to $C^2(t, r) = s(r)\phi(t)$ while for $\mu' = 0$, this yields

$$\left(-\epsilon - 4\pi\Pi + \frac{s^2}{C^4}\right)' + 3\frac{(-\epsilon - 4\pi\Pi + \frac{s^2}{C^4})C'}{C} = \frac{ss'}{C^4} + \frac{s^2C'}{C^5}. \quad (43)$$

Using Eqs.(4) and (20) in (41), it follows that

$$\begin{aligned} & \left(-\epsilon - 4\pi\Pi + \frac{s^2}{C^4}\right)' + 3\frac{(-\epsilon - 4\pi\Pi + \frac{s^2}{C^4})\dot{C}}{C} \\ &= 4\pi(\mu + P_r)\sigma A - \frac{8\pi\Pi\dot{C}}{C} + \frac{s^2\dot{C}}{C^5}. \end{aligned} \quad (44)$$

These two equations represent evolution equations for the quantity $-(\varepsilon + 4\pi\Pi) + \frac{s^2}{C^4}$ which represents one of the structure scalars X_{TF} .

The tensor $X_{\alpha\beta}$ is defined as

$$X_{\alpha\beta} = {}^*R_{\alpha\gamma\beta\delta}^* = \frac{1}{2}\eta_{\alpha\gamma}{}^{\varepsilon\rho}R_{\varepsilon\rho\beta\delta}^*V^\gamma V^\delta, \quad (45)$$

where $R_{\alpha\beta\gamma\delta}^* = \frac{1}{2}\eta_{\varepsilon\rho\gamma\delta}R_{\alpha\beta}{}^{\varepsilon\rho}$ and $\eta_{\varepsilon\rho\gamma\delta}$ denote the Levi-Civita tensor. Tensor $X_{\alpha\beta}$ can be expressed through its trace and trace free part as

$$X_{\alpha\beta} = \frac{1}{3}X_T h_{\alpha\beta} + X_{TF}(\chi_\alpha\chi_\beta - \frac{1}{3}h_{\alpha\beta}). \quad (46)$$

One can find X_{TF} with the help of the field equations, Eq.(46) and (22)-(24) (see for detail [28]) as

$$X_{TF} = -\epsilon - 4\pi\Pi + \frac{s^2}{C^4}. \quad (47)$$

Thus the evolution equations (43) and (44) in terms of X_{TF} turn out to be

$$\begin{aligned} X'_{TF} + 3\frac{X_{TF}C'}{C} &= \frac{s^2C'}{C^5} + \frac{ss'}{C^4}, \\ \dot{X}_{TF} + 3\frac{X_{TF}\dot{C}}{C} &= 4\pi(\mu + P_r)\sigma A - \frac{8\pi\Pi\dot{C}}{C} + \frac{s^2\dot{C}}{C^5}. \end{aligned}$$

The corresponding solutions will be

$$X_{TF} = \frac{1}{C^3} \int_0^r (-8\pi\Pi C' C^2 + \frac{s^2 C'}{C^2} + \frac{ss'}{C}) dr, \quad (48)$$

$$X_{TF} = \frac{-4\pi}{C^3} \int_0^t [2\pi\dot{C} - (\mu + P_r)AC\sigma] C^2 dt - \frac{s^2}{C^4}. \quad (49)$$

The last equation represents that the anisotropy of pressure and electromagnetic field affect the initial state of the energy density.

4.4 Dissipative Geodesic Charged Dust

Here we consider the case of dissipative geodesic dust to explore the effect of dissipation on energy density inhomogeneity. For this purpose, we assume that dissipative dust is moving along geodesic, i.e., $P_r = P_\perp = 0$ and $A = 1$. The corresponding equations take the form

$$(\varepsilon - 4\pi\tilde{\mu})' + 3\frac{\dot{C}}{C} \left(\varepsilon - 4\pi\tilde{\mu} + \frac{2s^2}{3C^4} \right) - \frac{12\pi\tilde{q}C'}{BC} = 0, \quad (50)$$

$$(\varepsilon - 4\pi\tilde{\mu})' = -3\frac{C'}{C}\varepsilon - 2\frac{s^2 C'}{C^5} - \frac{12\pi\tilde{q}\dot{C}B}{C} + \frac{ss'}{C^4}. \quad (51)$$

Equation (51) leads to a quantity Ψ

$$\Psi = \varepsilon + \frac{12\pi}{C^3} \int_0^r \tilde{q}\dot{C}BC^2 dr. \quad (52)$$

Equation (51) implies that for $\Psi = s = 0$, we get $\tilde{\mu}' = 0$ similar to the uncharged case. For $\Psi = \tilde{\mu}' = 0$, we obtain a relation between s and C from (51) as discussed in the previous cases.

Taking $\tilde{\mu}' = 0$ in Eq.(51), it follows that

$$\Psi = \frac{1}{C^3} \int_0^r (\frac{ss'}{C} - 2\frac{s^2 C'}{C}) dr. \quad (53)$$

Using Eqs.(50), (4) and (20), we obtain the evolution equation of Ψ

$$\dot{\Psi} - \frac{\dot{\Omega}}{C^3} = -4\pi\tilde{\mu}\sigma - \frac{4\pi\tilde{q}'}{B} + \frac{4\pi\tilde{q}C'}{BC} + \frac{2s^2\dot{C}}{C^5}, \quad (54)$$

where

$$\Omega = 12\pi \int_0^r \tilde{q} \dot{C} B C^2 dr. \quad (55)$$

The general solution of Eq.(54) is

$$\Psi = \frac{1}{C^3} \int_0^t (\dot{\Omega} - 4\pi\sigma\tilde{\mu}C^3 - \frac{4\pi\tilde{q}'C^3}{B} + \frac{4\pi\tilde{q}C^2C'}{B} + \frac{2s^2\dot{C}}{C^2}) dt. \quad (56)$$

This shows that charged dust density inhomogeneity depends on dissipative terms, shear and the electromagnetic field, for the initially homogeneous configuration. To investigate the effect of these factors on evolution of Ψ , we consider shearfree case for which Eq.(4) implies that $C = Br$. Consequently, Eq.(55) yields

$$\Omega = 12\pi \int_0^r \frac{\tilde{q} \dot{C} C^3}{r} dr. \quad (57)$$

The corresponding evolution equation is

$$\Psi = \frac{1}{C^3} \int_0^t (\dot{\Omega} - 4\pi\tilde{q}'C^2r + 4\pi\tilde{q}CrC' + \frac{2s^2\dot{C}}{C^2}) dt. \quad (58)$$

Finally, we discuss the relaxation effects in evolution of Ψ . The diffusion approximation ($\epsilon = 0$) implies $\tilde{q} = q$, $\tilde{\mu} = \mu$. Thus from Eq.(21), it follows that

$$\dot{q} = -\frac{4q}{3}\Theta + \frac{2ss'}{4\pi BC^4}. \quad (59)$$

Combining the above equation with transport equation (32), we have

$$q = -\frac{\kappa T'}{B(1 - \frac{4}{3}\Theta\tau)} - \frac{2ss'\tau}{4\pi BC^4(1 - \frac{4}{3}\Theta\tau)}. \quad (60)$$

For the shearfree case, this reduces to

$$q = -\frac{\kappa r T'}{C(1 - \frac{4}{3}\Theta\tau)} - \frac{2ss'r\tau}{4\pi C^5(1 - \frac{4}{3}\Theta\tau)}. \quad (61)$$

When we insert this value of q in Eq.(58), we obtain Ψ in terms of relaxation time τ which help to analyze relaxation effects in terms of electromagnetic field.

5 Conclusion

In this paper, we have studied how charge affects the energy density inhomogeneity and stability of the conformal flatness. For this purpose, the evolution equations of the Weyl tensor are formulated. We have investigated different aspects of fluid distribution responsible for energy density inhomogeneity. The main results are summarized as follows:

- In the case of non-dissipative charged dust and isotropic fluid, we have found that when we take electromagnetic field contribution, the energy density inhomogeneity is not controlled by the Weyl tensor alone but it also depends on charge. It is worth mentioning here that charge affects the conformal flat condition. Here the vanishing of the Weyl tensor implies that shear depends on charge. For shearfree fluid, the Weyl tensor disappears but this is not true for isotropic fluid. In the absence of charge, our result reduce to [24].
- For anisotropic fluid, it is found that the shear inhomogeneity also depends on charge in addition to anisotropy of pressure. A specific quantity associated with energy density inhomogeneity is identified as one of the structure scalar.
- Finally, we have investigated the effect of charge on dissipative geodesic dust. The effect of charge and different factors on the evolution of Ψ (a quantity that determines the existence of energy density inhomogeneity) are explored. The relaxation effects in the evolution of Ψ with the inclusion of charge are also indicated in this case.

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